Waves & Acoustics

Lecture-I

2nd Semester Hons.

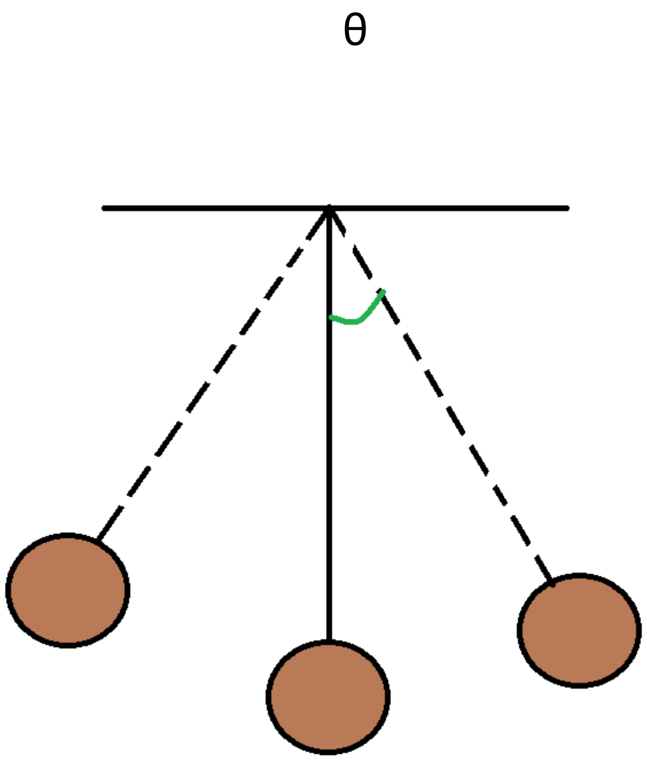
Paper- C4T

Department of Physics

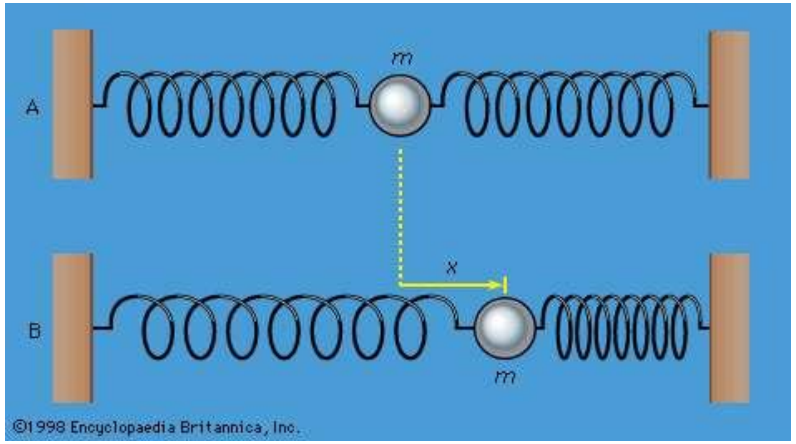
SMHGGDCW

Overview Of SHM

- **Periodic & Oscillatory Motion:-**
- The motion in which repeats after a regular interval of time is called periodic motion.
- The periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point called equilibrium position/mean position, is called oscillatory motion.
- In all type of oscillatory motion one thing is common i.e each body (performing oscillatory motion) is subjected to a restoring force that increases with increase in displacement from mean position.



Types of oscillatory motion :-



It is of two types such as linear oscillation and circular oscillation.

Example of linear oscillation:-

1. Oscillation of mass spring system.
2. Oscillation of fluid column in a U-tube.
3. Oscillation of floating cylinder.
4. Oscillation of body dropped in a tunnel along earth diameter.
5. Oscillation of strings of musical instruments.

Example of circular oscillation :-

1. Oscillation of simple pendulum.
2. Oscillation of solid sphere in a cylinder (If solid sphere rolls without slipping).
3. Oscillation of a circular ring suspended on a nail.
4. Oscillation of balance wheel of a clock.
5. Rotation of the earth around the sun.

Linearity

An important aspect of linear systems is that the solutions obey the Principle of Superposition , that is, for the superposition of different oscillatory modes, the amplitudes add linearly . The linearly-damped linear oscillator is an example of a linear system that involves only linear operators , that is, it can be written in the operator form as follows -

$$\left(\frac{d^2}{dt^2} + \Gamma\frac{d}{dt} + \omega_0^2\right)x(t) = A\cos(\omega t)$$

Where ω_0 represents the natural frequency of oscillation and Γ is regarded in general as the damping factor which we will discuss later.

The quantity in the brackets on the left hand side is a linear operator that can be designated by L where

$$Lx(t)=F(t)$$

Properties of linear operators

An important feature of linear operators is that they obey the principle of superposition. This property results from the fact that linear operators are distributive, that is

$$L(x_1 + x_2) = L(x_1) + L(x_2)$$

Therefore if there are two solutions $x_1(t)$ and $x_2(t)$ for two different forcing functions $F_1(t)$ and $F_2(t)$

$$Lx_1(t) = F_1(t)$$

$$Lx_2(t) = F_2(t)$$

then the addition of these two solutions, with arbitrary constants, is also a solution for linear operators.

$$L(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 L(x_1) + \alpha_2 L(x_2)$$

Superposition Principle

- If $\mathbf{Y_1}$ and $\mathbf{Y_2}$ represents two independent SHM

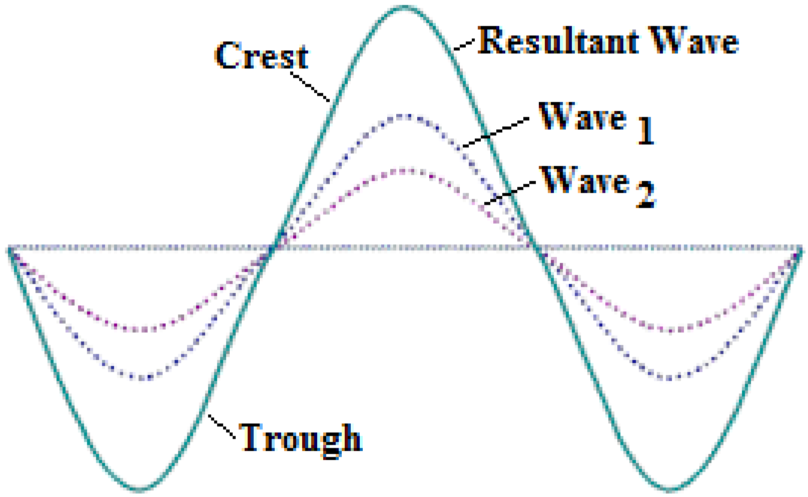
Then the Resultant of the above two SHM will be given as

$$Y = Y_1 + Y_2$$

This is known as Superposition Principle of waves.

If

$$Y_1 = A_1 \mathrm{Sin}(\omega_1 t) \quad \& \quad Y_2 = A_2 \mathrm{Sin}(\omega_2 t)$$



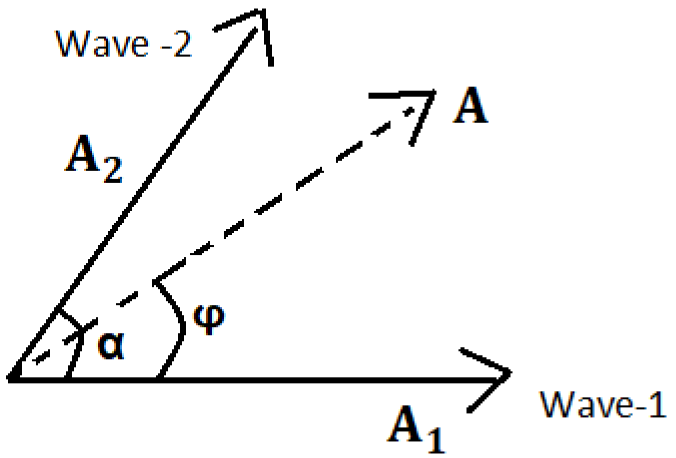
Then

$$Y = A_1 \mathrm{Sin}(\omega_1 t) + A_2 \mathrm{Sin}(\omega_2 t)$$

Superposition of Two Collinear Harmonic oscillations having equal frequencies

Two Harmonic Oscillations having equal frequencies and which are Collinear are represented as-

$$Y_1 = A_1 \mathrm{Sin}(\omega t) \quad \& \quad Y_2 = A_2 \mathrm{Sin}(\omega t + \alpha)$$



Where α is the phase difference between the two SHMs.
Then the resultant wave will be given as-

$$Y = A\ \mathrm{Sin}(\omega t + \varphi)$$

Where, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\mathrm{Cos}(\alpha)}$ is the resultant amplitude

and $\varphi = tan^{-1}\left(\frac{A_1\mathrm{Sin}(\alpha)}{A_1 + A_2\mathrm{Cos}(\alpha)}\right)$ Is the phase of the resultant wave with respect to $\mathbf{Y_1}$.

If the phase difference between the two waves is zero then the two Waves are said to be superposed in phase. And the resultant wave will look like as shown in Fig. 1.

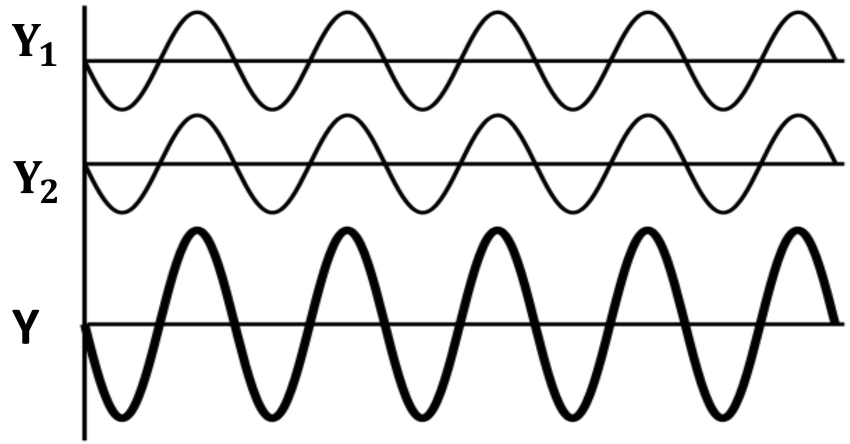


Fig. 1

The resultant amplitude will be given as-

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(0)} \Rightarrow A = A_1 + A_2$$

If the phase difference between the two waves is 180^o then the two Waves are said to be superposed out of phase . If the amplitude of the two waves are equal then the resultant wave will look like as shown in Fig. 2

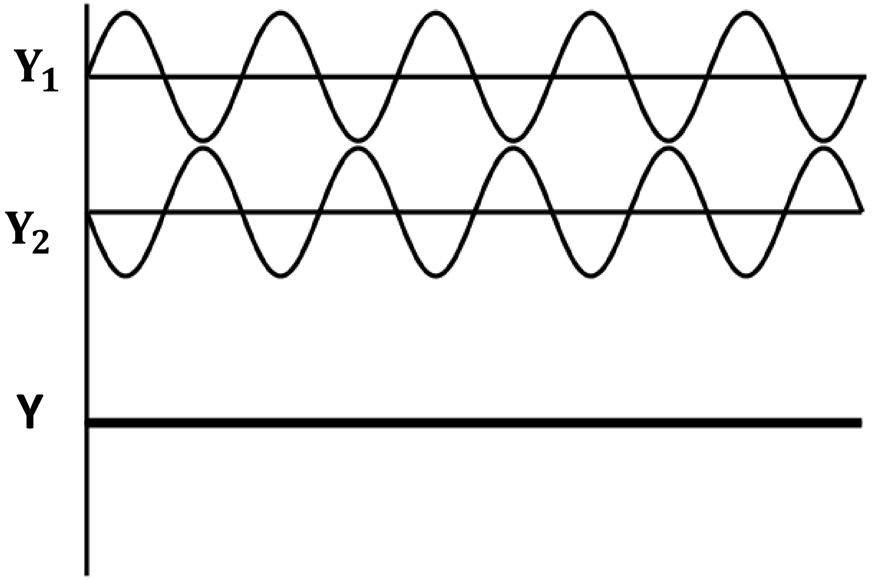


Fig. 2

The resultant amplitude will be given as-

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\pi)} \Rightarrow A = A_1 - A_2$$

$$= 0 \quad [\text{If } A_1 = A_2]$$

Superposition of Two Collinear Harmonic oscillations having unequal frequencies and formation of beats

Due to the superposition of two SHMs having same amplitude (for simplicity) and slightly different frequency with each other the amplitude of the resultant SHM changes from a minimum to a maximum value periodically .This is known as beats. Number of beats produce Per second is called beat frequency. Numerically beat frequency is equal to the difference in the frequencies of the interfering SHMs.

Let, the two interfering SHMs having slightly different frequency is represented as-

$$Y_1 = A\ \mathrm{Sin}(\omega_1 t) \quad \& \quad Y_2 = A\ \mathrm{Sin}(\omega_2 t)$$

Where $\omega_1 \sim \omega_2 = \Delta\omega$ is very small

Using Superposition principle the resultant Oscillation will be given as-

$$Y = Y_1 + Y_2$$

Therefore

$$Y = A(\sin(\omega_1 t) + \sin(\omega_2 t))$$

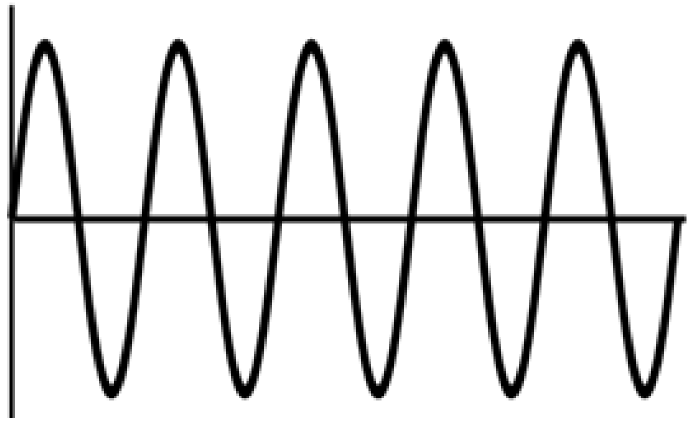
$$Y = 2A\sin\left(\frac{\omega_1 + \omega_2}{2}\right) t \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t$$

$$Y = A'\sin(\omega t)$$

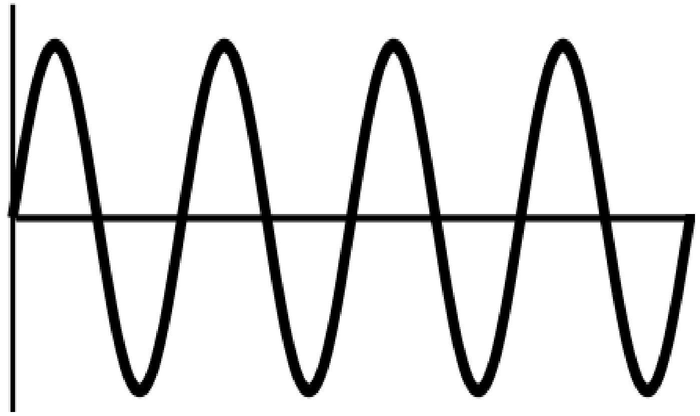
Where,

$$A' = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}\right) t$$

Y_1



Y_2



As we can see the amplitude of the resultant wave is not constant but it depends on time and the difference of the two interfering waves.

The amplitude becomes maximum when

$$\mathbf{Cos}\left(\frac{\omega_1-\omega_2}{2}\right)\mathbf{t}=\mathbf{1}$$

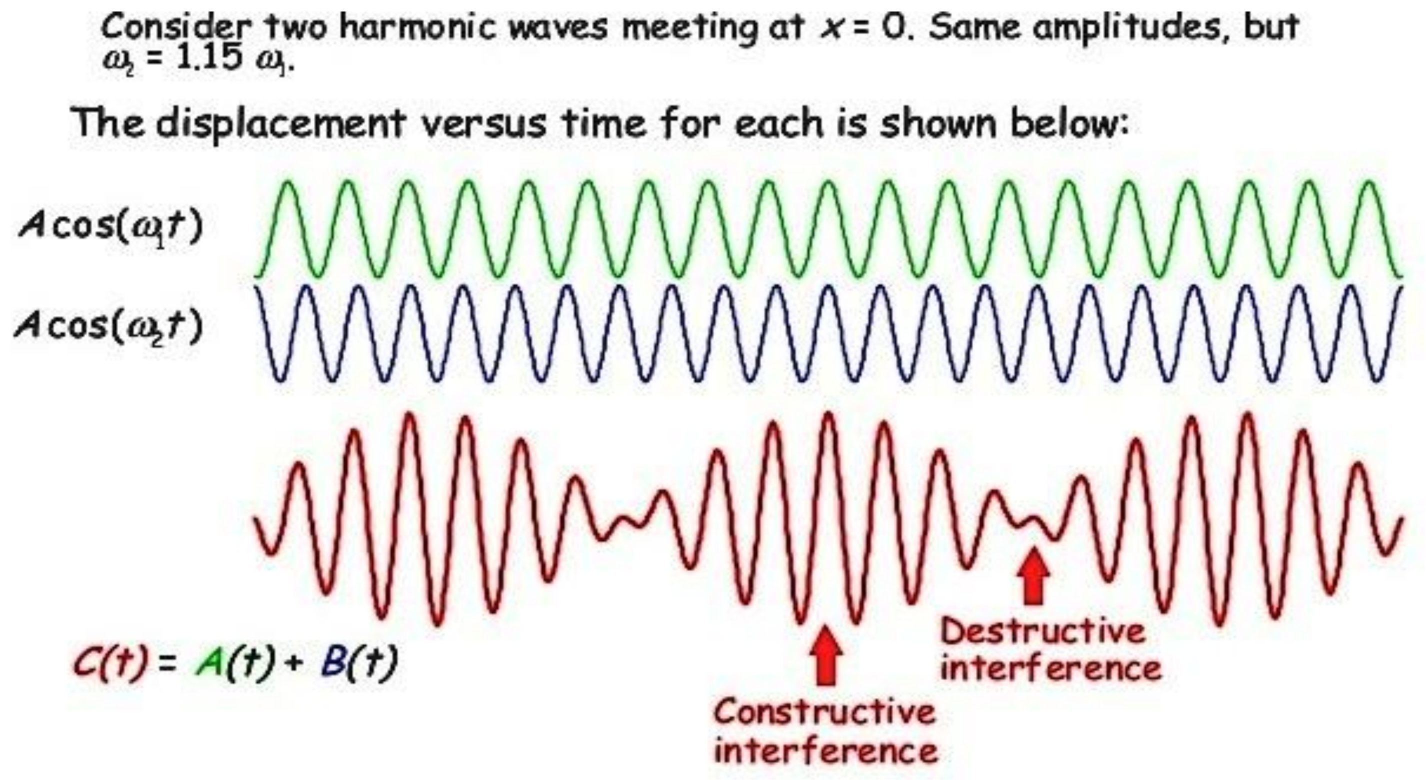
i.e. $\left(\frac{\omega_1-\omega_2}{2}\right)t = 2n\pi$

or, $$t=\frac{2n\pi}{\left(\frac{\omega_1-\omega_2}{2}\right)}$$

The amplitude becomes minimum when

$$\mathbf{Cos}\left(\frac{\omega_1-\omega_2}{2}\right)\mathbf{t}=\mathbf{0}$$

i.e. $\left(\frac{\omega_1-\omega_2}{2}\right)t = (2n+1)\pi/2$ ➡ $t=\frac{(2n+1)\pi/2}{\left(\frac{\omega_1-\omega_2}{2}\right)}$



The time difference two successive maximum or minimum is therefore

$$\Delta t=\frac{1}{n_1-n_2} \qquad [\omega=2\pi n]$$

And hence the beat frequency will be

$$\frac{1}{\Delta t}=n_1-n_2$$