Radioactivity

Nuclear Stability

Odd-Even nature of the nucleons:

Neutron (N)	Proton (Z)	Mass no (A)	No of stable nuclides	Abundance, % in the earth's cast	Examples
Even	Even	Even	~ 162	~ 85	$_{2}He^{4}$, $_{8}O^{16}$, $_{14}Si^{28}$, $_{20}Ca^{40}$, $_{26}Fe^{56}$, $_{82}Pb^{206}$
Even	Odd	Odd	~ 52	~ 13	$_{13}Al^{27}$, $_{11}Na^{23}$, $_{19}K^{39}$
Odd	Even	Odd	~ 50	~ 2	$_{8}O^{17}$, $_{12}Mg^{25}$
Odd	Odd	Even	~ 4	0	$_{1}H^{2}$, $_{3}Li^{6}$, $_{5}B^{10}$, $_{7}N^{14}$

(a) **Harkins's rule:** The nuclei having even number of neutrons and protons are the most stable and most abundant ones.

(b) For odd A, the number stable nuclides do not depend on whether N is odd or Z is odd.

(c) A proton pairs with only another proton and not with a neutron and same thing also occurs for neutron.

(d) Stability of α-particle nuclides and rule of Oddo: Nuclides having mass number as multiples of 4 are more abundant than their immediate neighbour. This illustrates the nuclear stability of 2-proton + 2-neutron combination. O^{16}, 20^{Ne}, Mg^{24}, Si^{28}, S^{32}, Ar^{36}, Ca^{40}

The neutron to proton ratio and nuclear stability

The neutron to proton ratio (n/p) depending on Z, is important to predict the stability of nuclides. For the elements up to Z = 20, the n/p ratio is unity and it increases gradually upto 1.6 for higher atomic number.

The neutron to proton ratio and different modes of decay:

Case I (higher n/p ratio): To decrease the value of n/p ratio the nuclide may decay

i) Beta emission: neutron converted to proton with emission of β^- radiation (electron).

$${}_0n^1 \longrightarrow {}_{+1}P^1 + {}_{-1}e^0 + {}_0\bar{\upsilon}^0$$

Examples:

$${}_6C^{14}\ (n/p = 1.33) \longrightarrow {}_7N^{14}\ (n/p = 1.00) + {}_{-1}e^0 + {}_0\bar{\upsilon}^0$$

(t½ = 18 min) (t½ = 3.7 hrs) (t½ = 28 days)

$$\begin{array}{lclclcl} {}_{56}Ba^{141} & \longrightarrow & {}_{57}La^{141} + {}_{-1}e^0 + {}_0\bar{\upsilon}^0 & \longrightarrow & {}_{58}Ce^{141} + {}_{-1}e^0 + {}_0\bar{\upsilon}^0 & \longrightarrow & {}_{59}Pr^{141} + {}_{-1}e^0 + {}_0\bar{\upsilon}^0 \\ n/p = 1.51 & & 1.47 & & 1.43 & & 1.39 \end{array}$$

(ii) Neutron emission: Through neutron emission n/p ratio can also decreased. Neutron has high nuclear binding energy (≈ 8 MeV), the neutron emission takes place only in high energetic nuclides.

$${}_{36}Kr^{87}\ (n/p = 1.41) \longrightarrow {}_{36}Kr^{86}\ (n/p = 1.39) + {}_0n^1$$

Case II (lower n/p ratio): To increase the value of n/p ratio the nuclide may decay

(a) **Positron emission**: Positrons are usually positive electrons and it leads to β+ radiation due to transformation of protons into neutrons. ${}_{+1}P^1 \longrightarrow {}_0n^1 + {}_{+1}e^0 + {}_0v^0$

$${}_{10}Ne^{19}\ (n/p = 0.9) \longrightarrow {}_9F^{19}\ (n/p = 1.11) + {}_{+1}e^0 + {}_0v^0$$

$${}_6C^{11}\ (n/p = 0.83) \longrightarrow {}_3B^{11}\ (n/p = 1.2) + {}_{+1}e^0 + {}_0v^0$$

(b) **K – electron capture**: When sufficient energy (>1.02 MeV) for positron emission is not available, the nucleus may capture an electron from the nearest shell i.e. K-shell and consequently one proton gets converted into neutron. $_{+1}P^{1} + _{-1}e^{0} \longrightarrow _{0}n^{1} + _{0}v^{0}$

$_{25}Mn^{54}$ (n/p = 1.16) + $_{-1}e^{0} \longrightarrow _{24}Cr^{54}$ (n/p = 1.25) + $_{0}v^{0}$

$_{23}V^{48}$ (n/p = 1.08) + $_{-1}e^{0} \longrightarrow _{22}Ti^{48}$ (n/p = 1.18) + $_{0}v^{0}$ or $_{23}V^{48} \longrightarrow _{22}Ti^{48} + _{+1}e^{0} + _{0}v^{0}$

[Due to the capture of an electron from K-shell, a vacancy is created. This vacancy is filled by an electron jumping from the higher shells. Such transition from the higher shells to the lower shells produces the characteristic X-rays.]

(c) **α-emission**: It generally occurs for the heavy nuclides.

$_{92}U^{238}$ (n/p = 1.58) $\longrightarrow _{90}Th^{232}$ (n/p = 1.60) + $_{2}He^{4}$

(d) **Proton emission**: Proton emission can also increase the n/p ratio, but it will require an additional amount of energy (compared to neutron) to cross the Coulombic potential barrier as it is a charged particle. Hence, this mode of decay occurs for the nuclides of very much energy.

Packing fraction and nuclear stability:

Aston notice that nuclear mass obtained from mass spectrograph is slightly deviate from the mass number (sum of protons and neutrons). This deviation was expressed in terms of packing fraction defined as

$$\text{Packing fraction} = \frac{\text{nuclear mass - mass number}}{\text{mass number}} \times 10^{4}$$

Here, nuclear mass is equal to atomic mass ignoring the mass of electron and the mass number is generally the nearest whole number to the atomic mass. The factor is arbitrary.

Example: $_{28}Ni^{58}$ whose mass is 57.942. So packing fraction is

$$\text{Packing fraction} = \frac{57.942 - 58}{58} \text{ X } 10^4 = -10$$

It is worth mentioning that the actual mass of the nucleons is always greater than the mass number. Mass of the proton = 1.0076 and neutron = 1.0089. For $_{28}Ni^{58}$, actual nuclear mass = (1.0076x28) + (1.0089x20) = 58.4798. So actual packing fraction should be defined as

$$\text{Packing fraction} = \frac{\text{nuclear mass - actual mass of nucleons}}{\text{mass number}} \text{ X } 10^4$$

- Packing fraction may be negative, positive and zero
- Negative packing fraction means that there is loss of mass which is converted into energy to bind the nucleons. Thus, the negative value indicates stable nuclides.
- Positive and zero packing fraction mean that unstable or less stable nuclides.

Significance of Packing fraction:

i) Prediction of stability region in terms of A: From the packing fraction curve it is seen that elements of mass no. near 50 have higher negative packing fraction value. Such nuclei are most stable and highly abundant.

ii) Prediction of fission and fusion: The nuclides having positive or less negative packing fraction tend to move more negative packing fraction. It may attained by fission by heavier elements and fusion by lighter elements.

iii) Interpretation of the Oddo rule: The bifurcation of packing fraction curve indicates that nuclides with mass number of multiple integral of helium are relatively stable.

Mass defect and nuclear binding energy

The difference between the theoretically calculated atomic mass and measured atomic mass is called mass defect. The theoretically calculated atomic mass is the sum of the mass of the constituent particle i.e. neutron, proton and electron.

The lost mass is converted to energy according to the Einstein mass-energy equation ($E=mc^2$). 1 amu = 1/12 mass of a 12-C atom. $12/12 x 6.023 x 10^{23} = 1.66 x 10^{-27}$ kg

1 amu = $1.66 x 10^{-27}$ kg x $(3x10^8 \text{ ms}^{-1})^2 = 14.92x10^{-11}$ J = 931 MeV

And 1 MeV = 10^6 eV = $10^6 x 1.6x10^{-19}$C x 1V = $1.6x10^{-13}$ J

Mass defect (Δm) and nuclear binding energy (NBE) is related as, NBE = Δm x 931 MeV

The average binding energy (B) is the nuclear binding energy per nucleon. It is always positive whether the nucleus is stable or not. It indicates that during construction of the nucleus some mass is always converted to energy to bind the nucleons within the nucleus.

The variation of B with mass number curve is in fact reciprocal of packing fraction curve.

Significance of the nuclear binding energy curve:

i) The lighter nuclides with mass number integral multiple of four are very stable.

ii) Medium nuclides have maximum nuclear binding energy and most abundant.

iii) In Heavy nuclides nuclear binding energy falls down and undergoes nuclear fission.

Problem:

The maximum kinetic energy of positrons emitted by ${}_{7}N^{13}$ is 1.20 MeV. Calculate the mass of the ${}_{7}N^{13}$ nuclide (m_N) from the masses of ${}_{6}C^{13}$ (m_C = 13.00335 u) and mass of electron (m_e = 0.00055 u)

The conversion can be written as: ${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C}^{-} + {}^{0}_{+1}e + {}^{0}_{0}\upsilon \rightarrow {}^{13}_{6}C + {}^{0}_{-1}e + {}^{0}_{+1}e + {}^{0}_{0}\upsilon$

Due to conversion of proton to neutron C- is formed momentarily which have one extranuclear electron and ultimately it is converted to C through loosing this electron to maintain electrical neutrality. Here the resultant mass loss is converted to kinetic energy of positron. So, Δm = 1.20 MeV = (1.20/931) u = 0.00129 u

$\Delta m = m_N - m_C - 2m_e$ (mass of positron = mass of electron)

$\Rightarrow m_N = \Delta m + m_C + 2m_e$ = 0.00129 +13.00335 + (2 x 0.00055) = 13.00574 u

Problem:

Calculate the maximum energy of a β particle from a 14C nucleus which does not emit any γ radiations. The masses of ${}^{14}C$ and ${}^{14}N$ isotopes are 14.003242 u and 14.003074 u respectively.

The maximum energy of the β-particle may be calculated from the supplied mass difference assuming that no energy is shared by the accompanying antineutrino.

The conversion can be written as:

$${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N}^{+} + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} \rightarrow {}^{14}_{7}N - {}^{0}_{-1}e + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon}$$

Due to conversion of neutron to proton, N^+ is formed momentarily which have one extra positive charge and ultimately it is converted to N by accepting one electron to maintain electrical neutrality. Here the resultant mass loss is converted to kinetic energy of β particle.

So, Δm = mass of ${}^{14}C$ – mass of ${}^{14}N$ = (14.003242 – 14.003074) u = 0.000168 u

Therefore, energy released in the process = 0.000168 x 931 MeV = 0.3554 MeV
This is the maximum energy of a β-particle.

Nuclear Forces: Meson theory

Japanese physicist Hideki Yukawa proposed the existence of π-meson. These are three type π^+, π^- (charged) and π^0 (neutral) meson. Each of π^+ and π^- has rest mass ~ $273m_e$ and π^0 meson is having the rest mass ~$264m_e$. They have no spin. According to Yukawa, nucleons are inter converted very fast within the nucleus as follows, $n \rightleftharpoons p + \pi^-$; $p \rightleftharpoons n + \pi^+$;

$n \rightleftharpoons n + \pi^0$; $p \rightleftharpoons p + \pi^0$

So, the exchange of charged meson explains the binding energy of neighbouring protons and neutrons. Similarly, neutral meson explains the binding energy of same kind nucleons. All the conversions processes are in dynamic equilibrium in a stable nucleus. If any type of nucleons is increased then this equilibrium will be lost and the nucleus becomes unstable. This is why for a particular number of protons, there should be a fixed n/p ratio.

Nuclear Shell Model: Magic Number

Need of Shell Model:

1. For detailed properties of nuclei

2. To study about the ‘l’ and ‘s’ effect on nucleons

3. Magic number: 2, 8, 20, 28, 50, 82, 126

The nuclei having magic number of proton or neutron are stable than the neighbours nuclei.

Essential features in the construction of the nuclear shell model:

1) Independent particle model: The electron shell model is not used for nuclear shell model because in case of electron shell model electron are revolving around the nucleus, so there is a central object but in nuclear shell model the nucleons are moved under the attractive force of all other nucleons.

2) Separate shell for each kind of nucleons: Protons and neutrons are moving in separate shell. So, there are separate configuration for protons and neutrons. Another fact is that, in atom most of the space is vacant that means electrons are revolving in an empty space but nucleons have to move through a densely packed nucleus.

3) Fermion nature of the nucleons: No two nucleons of each kind can have all quantum numbers same in a nuclide. Thus, each can be filled with two nucleons with their opposite spins.

4) Orbital representation and energy sequence: It is explained on the basis of spin orbit interaction and harmonic oscillator. $E_n = (N+½) h\nu$ where N = 0, 1, 2, 3, and so on

If N is even, then ‘l’ (orbital angular momentum) = 0, 2, 4, 6, and so on.

And if N is odd, ‘l’ (orbital angular momentum) = 1, 3, 5, 7, and so on.

For l = 0, 1, 2, 3, 4,.... the orbitals are designated as s, p, d, f, g,....... respectively.

5) Spin orbit coupling and energy sequence: Strong spin orbit coupling for each kind of nucleon, total angular momentum j arises and it can have two value Levels having higher value of total angular momentum (j) will have lower energy. (l ± 1/2). The sub shell having j = (l + 1/2) has lower energy compared to j= (l - 1/2). Each orbital can accommodate (2j+1) number of nucleons. Hund's rule of spin multiplicity is not maintained. Two nucleons are paired always.

N	l	State without LS coupling	State with j j = (l±s) (always s = ½)	No of nucleons in j state (2j+1)	No of nucleons in shell	
0	0	1s	l = 0 and s = ½ and j = ½ $1s_{½}$	2	2	2
1	1	1p	1 = 1 and s = ½ and j = 3/2, 1/2 $1p_{3/2}$, $1p_{1/2}$	$1p_{3/2}$ = 4 $1p_{1/2}$ = 2	6	2+6=8
2	2 0	1d 2s	1 = 2 and s = ½ and j = 5/2, 3/2 $1d_{5/2}$, $1d_{3/2}$ l = 0 and s = ½ and j = ½ $2s_{½}$	$1d_{5/2}$ = 6 $1d_{3/2}$ = 4 $2s_{½}$ = 2	12	2+6+12 =20
3	3 1	1f 2p	1 = 3 and s = ½ and j = 7/2, 5/2 $1f_{7/2}$, $1f_{5/2}$ 1 = 1 and s = ½ and j = 3/2, 1/2 $2p_{3/2}$, $2p_{1/2}$	$1f_{7/2}$ = 8 $1f_{5/2}$ = 6 $2p_{3/2}$ = 4 $2p_{1/2}$ = 2	20	
4	4 2 0	1g 2d 3s	1 = 4 and s = ½ and j = 9/2, 7/2 $1g_{9/2}$, $1g_{7/2}$ 1 = 2 and s = ½ and j = 5/2, 3/2 $2d_{5/2}$, $2d_{3/2}$ 1 = 0 and s = ½ and j = 1/21 $3s_{1/2}$	$1g_{9/2}$ = 10 $1g_{7/2}$ = 8 $2d_{5/2}$ = 6 $2d_{3/2}$ = 4 $3s_{1/2}$ = 2	30	

Nuclear Configuration: $1s_{½}^{2}$, $1p_{3/2}^{4}$, $1p_{1/2}^{2}$, $1d_{5/2}^{6}$, $2s_{½}^{2}$, $1d_{3/2}^{4}$, $1f_{7/2}^{8}$, $1p_{3/2}^{4}$, $1f_{5/2}^{6}$, $1p_{1/2}^{2}$, $1g_{9/2}^{10}$, $1g_{7/2}^{8}$, $2d_{5/2}^{6}$.................

Determination of spin and parity:

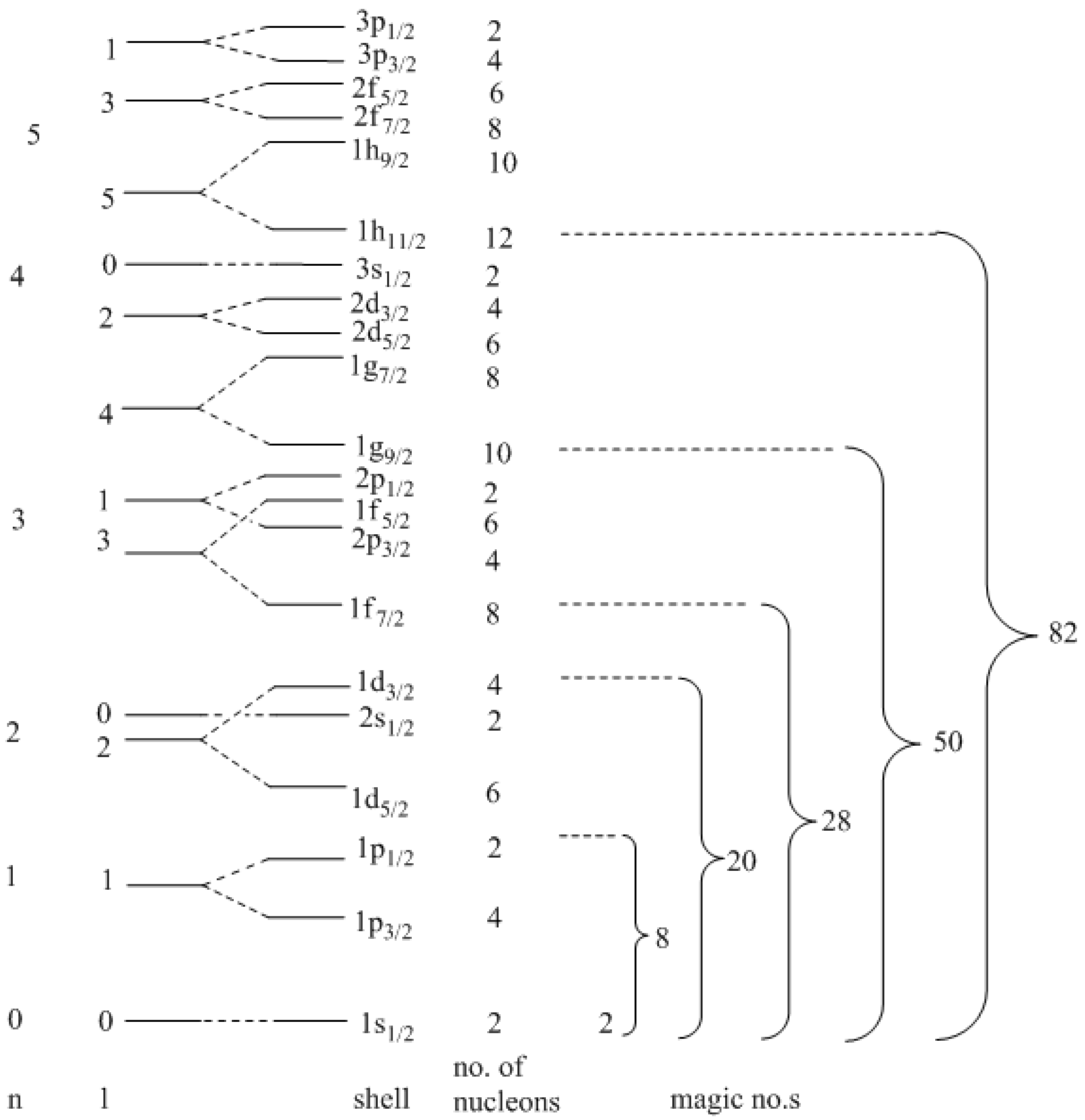
Nuclei having odd mass number:

In this case, only one kind of nucleons is remains unpaired and j of that sub shell gives the value of nuclear spin. The paired nucleons contribute nothing in determining spin.

A nucleon is of even parity means its orbital is symmetric with respect to its sign while odd parity means its orbital is not symmetric with respect to sign. Thus, l = 0, 2, 4, 6 (i.e. s, d, g etc) denote even (+) parity and l = 1, 3, 5 (i.e. p, f, h etc) will have odd (–) parity.

Nuclei having odd number of nucleons for each kind:

In this case, one proton and one neutron remain unpaired separately. Momenta of the nucleons interact to determine the spin and parity. Here, Nordheim Rules are important. If l_p and l_n are the orbital angular momentum for the unpaired proton and neutron respectively and j_p and j_n are their corresponding total angular momentum then I, will be $I = |j_p - j_n|$ when $(l_p + l_n + j_p + j_n)$ is even and $I = |j_p + j_n|$ when $(l_p + l_n + j_p + j_n)$ is odd. And parity is even (+) when l_p and l_n are both even or odd and parity is odd (-) when either l_p or l_n is odd.



Success of Shell Model:

1) Explain the existence of Magic Numbers.

2) Explain ground and lower excitation states of the nucleus.

3) Gives the values of nuclear spin and parity.

4) Explain the extra stability and high binding energy.

5) Tendency of pairing among the nucleons of each kind.

6) Abundance in nature

7) Number of stable isotope

8) Cross section of neutron capture

9) Tendency of α and β decay

Limitation of Shell Model:

1) Could not explain the correct value of total angular momentum (j) for heavy nuclei like $_{33}As^{75}$, $_{25}Mn^{55}$.

2) Could not explain the four stable nuclei $_1H^2$, $_3Li^6$, $_5B^{10}$, $_7N^{14}$

3) Could not explain the ground state odd nuclei 150 < A > 190, A > 220

4) Could not explain excited states of even nuclei

Q1. According to nuclear shell model, what is the protons configuration for the $_{13}Al^{27}$?

Number of proton = 13 and neutron = 14

$_{13}p = 1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^2, 1d_{5/2}^5, 2s_{1/2}, 1d_{3/2}$......and $_{14}n = 1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^2, 1d_{5/2}^6, 2s_{1/2}, 1d_{3/2}$.......

Q2. What is the ground state spin and parity of $_{13}Al^{27}$?

Number proton = 13 and neutron = 14
$_{13}p = 1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^2, 1d_{5/2}^5$ and $_{14}n = 1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^2, 1d_{5/2}^6$

So, for $_{13}Al^{27}$ nucleus, only $1d_{5/2}^5$ shell determines the spin and parity.

Total spin (j) = 5/2 and parity = $(-1)^l = (-1)^2 = +1$ (even) [for d, l = 2] So, parity $5/2^+$

Q3. In the nuclear shell model the spin parity of ^{15}N is given by (a) $1/2^-$ **(b) $3/2^+$ (c) $3/2^-$ (d) $1/2^+$**

Number proton = 7 and neutron = 8; $_7p = 1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^1$ and $_8n = 1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^2$

Only $1p_{1/2}^1$ shell determines the spin and parity.

Total spin (j) = 1/2 and parity = $(-1)^l = (-1)^1 = -1$ (odd) [for p, l = 1] So, spin parity $1/2^-$

Q4. According to the single particle nuclear shell model the spin parity of the ground state of $_8O^{17}$ is (a) $1/2^-$ (b) $3/2^-$ (c) $5/2^+$ (d) $3/2^+$

Number proton = 8 and neutron = 9; $_8p$ = $1s_{½}^2$, $1p_{3/2}^4$, $1p_{1/2}^2$ and $_9n$ = $1s_{½}^2$, $1p_{3/2}^4$, $1p_{1/2}^2$, $1d_{5/2}^1$

Only $1d_{5/2}^1$ shell contributes to the spin and parity.

Total spin (j) = 5/2 and parity = $(-1)^2 = (-1)^2 = +1$ (even) [for d, l = 2] So, spin parity $5/2^+$

Q5. What will be the spin and parity of $_4Be^9$ nucleus as predicted by the shell model?

Number proton = 4 and neutron = 5; $_4p$ = $1s_{½}^2$, $1p_{3/2}^2$ and $_5n$ = $1s_{½}^2$, $1p_{3/2}^3$

Only $1p_{3/2}^3$ shell contributes to the spin and parity.

Total spin (j) = 3/2 and parity = $(-1)^1 = (-1)^1 = -1$ (odd) [for p, l = 1] So, spin parity $3/2^-$

Q6. Determine the spin and parity of $_{33}As^{76}$?

Number proton = 33 and neutron = 43; $_{33}p$ = $1s_{½}^2$, $1p_{3/2}^4$, $1p_{1/2}^2$, $1d_{5/2}^6$, $2s_{½}^2$, $1d_{3/2}^4$, $1f_{7/2}^8$, $1p_{3/2}^4$, $1f_{5/2}^1$ and $_{43}n$ = $1s_{½}^2$, $1p_{3/2}^4$, $1p_{1/2}^2$, $1d_{5/2}^6$, $2s_{½}^2$, $1d_{3/2}^4$, $1f_{7/2}^8$, $1p_{3/2}^4$, $1f_{5/2}^6$, $1p_{1/2}^2$, $1g_{9/2}^3$ Both $1f_{5/2}^1$ and $1g_{9/2}^3$ shell contributes to the spin and parity.

Here, $(l_p + l_n + j_p + j_n) = (3 + 4 + 5/2 + 9/2) = 14$ (even) and $I = |j_p - j_n| = 9/2 - 5/2 = 2$

Total spin (j) = 2 and parity = odd (l_p, odd and l_n, even) So, spin and parity 2^-

Hybridisation of nucleons energy levels: Spin of $_9F^{19}$

When two energy levels are very close then nucleons of lower energy level may move to higher energy level. This is considered as hybridisation of nucleons.

In $_9F^{19}$, $_9p$ = $1s_{½}^2$, $1p_{3/2}^4$, $1p_{1/2}^2$, $1d_{5/2}^1$ and $_{10}n$ = $1s_{½}^2$, $1p_{3/2}^4$, $1p_{1/2}^2$, $1d_{5/2}^2$. So, only $1d_{5/2}^1$ determines the spin and parity. Spin and parity will be $5/2^+$. But, experimentally it is found to be $1/2^-$. It can be explain by considering the shifting of a proton from $1p_{1/2}$ to $1d_{5/2}$, that is the unpaired proton now in the $1p_{1/2}$ level. Therefore spin and parity will be $1/2^-$

Artificial radioactivity

Artificial radioactivity means production of radioactivity by an artificial way.

Transmutation of element

Transmutation of elements means conversion of an element into another element by some artificial way. By performing a nuclear reaction an element may convert to another element. Generally, a transmutation nuclear reaction is represented as follows:

${}_{Z1}X^{A1} + {}_{Z2}P^{A2} \rightarrow {}_{Z3}Y^{A3} + {}_{Z4}E^{A4} + Q$ where ${}_{Z1}X^{A1}$ (target nucleus), ${}_{Z2}P^{A2}$ (projectile), ${}_{Z3}Y^{A3}$ (product nuclide) and ${}_{Z4}E^{A4}$ (ejectile). A shorter notation is also used to represent the nuclear reaction as X^{A1} (P, E) Y^{A3} i.e. target nuclide (projectile, ejectile) product nuclide.

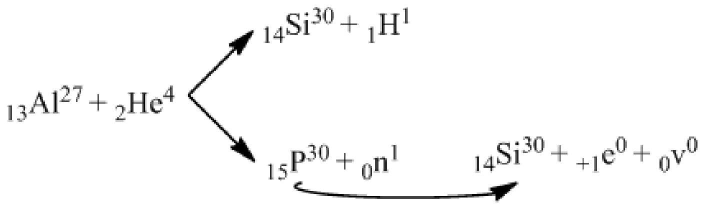
Examples:

${}_{5}B^{10} + {}_{2}He^{4} \rightarrow {}_{6}C^{13} + {}_{1}H^{1}$ i.e. B^{10} (α, p) C^{13}

${}_{7}N^{14} + {}_{2}He^{4} \rightarrow {}_{8}O^{17} + {}_{1}H^{1}$ i.e. N^{14} (α, p) O^{17}

${}_{9}F^{19} + {}_{2}He^{4} \rightarrow {}_{10}Ne^{22} + {}_{1}H^{1}$ i.e. F^{19} (α, p) Ne^{22}

${}_{4}Be^{9} + {}_{2}He^{4} \rightarrow {}_{6}C^{12} + {}_{0}n^{1}$ i.e. Be^{9} (α, n) C^{12}



Nuclear reaction versus Chemical reaction

(i) Orbital electrons vs. Nucleus (ii) Energy change (iii) Conservation of mass (iv) Effect of chemical state (v) Isotope effect (vi) Effect of external factor

Nuclear Fission: The heavier nuclei on being bombarded by a projectile splits into two lighter fragments of comparable size, this process is called nuclear fission.

${}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{92}U^{236*} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\ {}_{0}n^{1}$ + energy

Characteristic features of Fission:

(a) Mass distribution in fission fragments: The compound nucleus very often undergoes an asymmetric fission. The mass number of the lighter fission product lies in the range 85-104, while the heavier fragment covers the range 130-149. The distribution of mass number can be represented in the fission yield curve known as Bohr yield curve.

(b) Energy distribution in fission fragments:

If the compound nucleus is assumed to be at rest then according to the law of conservation of momentum $M_1u_1 = M_2u_2$; or $u_1/u_2 = M_2/M_1$ where M terms denote the masses and u term represent velocities of fragments. So, ratio of kinetic energies will be

$$\frac{E_1}{E_2} = \frac{1/2M_1u_1^2}{1/2M_2u_2^2} = \frac{1/2(M_2u_2)u_1}{1/2(M_1u_1)u_2} = \frac{M_2}{M_1}$$

Thus, kinetic energies and the masses of the fragments are inversely related.

(c) Emission of projectile:

Ejectile of the nuclear fission can be utilized as projectile. The neutrons as ejectile are not emitted at one instant. More than 99% are emitted instantaneously within 10^{-14} s called **prompt neutrons**. The neutron (1%) which are emitted in late from the fission product are called **delayed neutron.** These delayed neutrons play an important role in controlling the nuclear fission. The delayed neutrons are of low energy while the prompt neutrons are of high energy.

${}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{92}U^{236*} \longrightarrow {}_{57}La^{146} + {}_{35}Br^{87} + 3\ {}_{0}n^{1}$ (prompt neutrons)

${}_{35}Br^{87} \longrightarrow {}_{36}Kr^{87} + {}_{-1}e^{0} \longrightarrow {}_{36}Kr^{86} + {}_{0}n^{1}$ (delayed neutrons)

(d) β^- activity in the fission products:

The starting nuclide ($Z = 92$) requires a relatively higher n/p ration for its stability and this high ratio on being transmitted to the fission products of much lower atomic number. So the fission products have high n/p ratio that's why to gain stability by decreasing n/p ration through β^- decay.

(e) Energy release in fission:

The energy released per fission is ~ 200 MeV which is much greater than any ordinary nuclear reaction because fission can proceed in a chain reaction. Calculation of energy released in the fission process can be done two ways.

i) Mass defect method:

$${}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{42}Mo^{98} + {}_{54}Xe^{136} + 2\,{}_{0}n^{1} + 4\,{}_{-1}e^{0} + 4\,{}_{0}v^{-0}$$

Mass loss (Δm) = $[m\,(U^{235}) + m\,(n^{1})] - [m\,(Mo^{98}) + m\,(Xe^{136}) + 2m\,(n^{1})]$

$= 235.004 + 1.0086] - [97.906 + 135.907 + (2 \times 1.0086)]$
$= 236.0526 - 235.8302 = 0.2224$ amu $= 0.2224 \times 931$ MeV $= 207$ MeV

(In case of β- disintegration the product species will remain as a cation at the moment of production that's why mass of electrons not accounted)

ii) Nuclear binding energy method: For the fissionable nuclides nuclear binding energy per nucleon (B) lies in the range ~ 7.5 MeV while the fission products have ~ 8.5 MeV. Thus, in each fission energy released will be 1 MeV per nucleon. For ${}_{92}U^{235}$ it is $1 \times 235 = 235$ MeV.

Energetics of fission and condition for fissionability:

Fission energy (E_f): Energy obtained from fission due to mass loss.

Energy barrier (E_b): The energy required to separate the charged fission fragments arises due to Coulombic interaction and nuclear force among the nucleons of the fissionable nuclide.

Activation energy ($E^{\neq}$): This amount of energy is required for fission. It is equal to (E_b-E_f)

Excitation energy (E_{ex}): Energy gain by the compound nucleus which is equal to projectile binding energy in the compound nucleus (E_B) + kinetic energy of the projectile.

i) $E_f > E_b$ gives the condition of spontaneous fission.

ii) $E_f < E_b$ gives the condition of non spontaneous fission. But, if $\{(E_b - E_f) \approx E^{\neq}\}$ is less than the E_{ex} then spontaneous fission occur.

It is important to note that the nuclides having odd number of neutrons show a relatively higher value of E_{ex} and reaction cross section. Probably, on pairing a neutron in the compound nucleus, a large amount of binding energy is released. For the nuclides already having even number of neutrons, the process is not favoured. For U^{235}, released binding energy ($E_B \approx E_{ex}$, 6.4 MeV) is greater than the activation energy ($E^{\neq}$, 5.4 MeV). This is why, U^{235} undergoes fission on the capture of a slow neutron even when neutron have negligible amount of energy. Thus both slow (< 1 eV) and fast ($\geq$ 1 MeV) neutron can affect the fission of U^{235}. But, fission cross section for the slow neutron is higher (580) than the fast neutron (1.27). However, for U^{238} to undergo fission, it will require at least ($E^{\neq}$, 5.9 – E_{ex}, 5.2) = 0.7 MeV energy from the projectile. But here cross section is very small. The fission cross section becomes appreciable only when the neutron energy exceeds 1.2 MeV that is fast neutron.

iii) $E_f << E_b$ the nuclides cannot undergo spontaneous fission. Fission can be induced only by the high energy projectile but with a low cross section.

Fertile and Fissile Nuclide

${}_{92}U^{233}$, ${}_{92}U^{235}$, ${}_{94}Pu^{239}$ these nuclides undergo fission on the other hand U^{238} and Th^{232} can never undergo fission but they can produce fissile nuclides. These nuclides are called fertile nuclides.

${}_{92}U^{235}$: The naturally occurring uranium, ${}_{92}U^{238}$, contains only negligible fraction of U^{235} and bulk fraction is U^{238}. There are several techniques such as **diffusion method**, thermal diffusion method used to enrich the fraction of U^{235}.

${}_{92}U^{233}$: It is produced from Th^{232} as follows

$${}_{90}Th^{232} + {}_{0}n^{1} \xrightarrow{\text{resonance capture}} {}_{90}Th^{*233} \xrightarrow[(t_{1/2} = 23.5\text{min})]{{}_{-1}e^{0}} {}_{91}Pa^{233} \xrightarrow[(t_{1/2} = 27.5\text{ d})]{{}_{-1}e^{0}} {}_{92}U^{233}$$

${}_{94}Pu^{239}$: It is produced from U^{238} as follows

$$\underset{\text{(slow)}}{{}_{92}U^{238} + {}_{0}n^{1}} \xrightarrow{\text{resonance capture}} {}_{92}U^{*239} \xrightarrow[(t_{1/2} = 23.5\text{min})]{{}_{-1}e^{0}} {}_{93}Np^{239} \xrightarrow[(t_{1/2} = 2.3\text{ d})]{{}_{-1}e^{0}} {}_{94}Pu^{239}$$

Nuclear Fusion:

At high temperature ($10^7 - 10^8$ K), the lighter nuclides may undergo combination or fusion to form relatively heavier and stable nuclides. Since, this reaction is often called thermonuclear reactions as it can only occur at a very high temperature.

$4\ {}_1H^1 \longrightarrow {}_2He^4 + 2\ {}_{+1}e^0$; $2\ {}_1H^2 \longrightarrow {}_2He^4$

Spallation:

When a target nuclide bombard with the very high energy projectile then the target nuclide breaks into large number of particles along with a product nuclei having atomic and mass number about 10-20 unit less than the target nuclei. Such reactions have been termed as spallation reactions.

${}_{33}As^{75} + {}_1H^2 = {}_{25}Mn^{56} + 9\ {}_1H^1 + 12\ {}_0n^1$

${}_{92}U^{238} + {}_2He^4 = {}_{74}W^{187} + 20\ {}_1H^1 + 35\ {}_0n^1$

${}_{29}Cu^{63} + {}_1H^1 = {}_{11}Na^{24} + 9\ {}_2He^4 + {}_1H^1 + 3\ {}_0n^1$

Radiocarbon dating:

The neutrons present in the cosmic rays can carry out the nuclear transmutation reaction, $^{14}N(n, p)^{14}C$ in atmosphere to generate the radiocarbon ^{14}C having $t_{½} = 5730$ yr.

$$^{14}_{7}N + ^{1}_{0}n = ^{14}_{6}C + ^{1}_{1}H; \quad ^{14}_{6}C \xrightarrow{t_{½} = 5730\,yr} ^{14}_{6}C + \beta + \bar{\upsilon}$$

The produced 14C actually exists as $^{14}CO_2$. Formation and decay of ^{14}C leads to an equilibrium mass ratio $^{12}CO_2 : ^{14}CO_2 = 1:10^{-12}$. The ratio is maintained as long as the plant is living. But when the plant died, the accumulated ^{14}C in the plant decreased due to radioactive disintegration. The fresh sample collected from various parts of the world show almost a constant value which is ~15.5±0.3 disintegrations per gram of total carbon per minute. From the knowledge specific gravity of any archaeological object (i.e. wood, cloth, charcoal) containing carbon, the age of the sample can be calculated by using relationship:

$$\text{Age} = \frac{2.303}{\lambda} \log\frac{\text{original activity}}{\text{final activity}}\ \text{yr}$$

$$= \frac{2.303 \times 5730}{0.693} \log\frac{\text{original activity}}{\text{final activity}}\ \text{yr}$$

Here original activity ~15.5±0.3 and final activity means activity at any time after plant died.

Example: An archaeological specimen containing ^{14}C gives 40 counts in 5 minutes per gram of carbon. A specimen of freshly cut wood gives 20.3 counts per gram of carbon per minute. The counter used recorded a back ground count of 5 counts per minute in absence of any ^{14}C containing sample. What is the age of the specimen?

Solution: The background counting rate is to be subtracted from the observed counting rate.

Hence, count rate of specimen = (40/5) – 5 counts min-1 = 3 counts min-1.

Count rate of fresh wood = (20.3 – 5) counts min-1. = 15.3 counts min-1.

Decay constant of 14C = 0.693/5730 yr. If t be the age of the specimen, then

$$t = \frac{2.303 \times 5730}{0.693} \log\frac{15.3}{3}\ \text{yr} = 13474\ \text{yr}$$

Application of radioactivity:

1) Geochemical and radio carbon dating: These are based on the rate at which a selected radioactive nuclide undergoes decay. As for example, the age of a uranium mineral may be determined from its lead-uranium ratio. This gives the minimum age of the earth.

The plants or animals intake carbon dioxide as $^{14}CO_2$, which is produced in the environment through bombardment of nitrogen by neutrons of cosmic rays. As soon as plant dies, the radioactive C-14 undergoes continuous decay. Therefore, age of the plant can be calculated by measuring the recent activity and constant original activity of C-14.

2) Tracer studies by radioisotopes: Since radioisotopes has the same chemical properties as the stable isotopes of the element, a trace quantity of the radioisotope mixed with the stable isotope may be inserted into the system under investigation without interfering chemical nature. The presence of radioisotope in different parts of the system may be traced with suitable counter and mode of progress of the system may be understood. This is the basis of the tracer methods which have been widely used in chemistry, biology, agriculture, medicine and metallurgy.

i) Structure and mechanism: Tracer study offer very satisfactory methods for the elucidation of structure of the different compounds and verification of mechanism of various reactions.

(a) Tracer study with ^{35}S has been used to establish the non-equivalence of the two sulfur atoms in the thiosulphate ion. Inactive sulphite ion is treated with ^{35}S (radioactive) and the resulting thiosulphate ion is decomposed by acid. It appears that the precipitated sulphur carries all the radioactivity, i.e, ^{35}S is not distributed within thiosulphate ion. S* radioactive

$$SO_3^{2-} + S^* \rightarrow S^*SO_3^{2-}; \qquad S^*SO_3^{2-} + 2H^+ \rightarrow H_2SO_3 + S^*$$

(b) The iodine atoms in the HgI_4^{2-} ion are equivalent as shown by tracer studies with radioactive iodine (I*)

$HgI_4 + 2NaI \rightarrow Na_2HgI_4^*$; $Na_2HgI_4^* + 2AgNO_3 \rightarrow 2AgI^* + HgI_4^* + 2NaNO_3$

The radioactive iodine is distributed uniformly among the decomposition products.

(c) Mechanism of reactions:

$[Co(NH_3)_5Cl]^{2+} + [Cr(H_2O)_6]^{2+} + 5H_2O \rightarrow [Co(H_2O)_6]^{2+} + [Cr(H_2O)_5Cl]^{2+} + 5NH_3$

Cr(II) is oxidized to Cr(III) while Co(III) is reduced to Co(II). By using radioactive chlorine in $[Co(NH_3)_5Cl]^{2+}$, it has been observed that the product Cr(III) always contains the radioactive chlorine. This has been explained by suggesting that the chloride ion of the Co(III) complex first forms a bridge between the chromium and cobalt atoms and facilitates the transfer of electron from Co(III) to Cr(II). The intermediate will have the structure $[(NH_3)_5Co\text{-}Cl\text{-}Cr(H_2O)_5]^{4+}$. Once, the electron is transferred, Cr(III) attracts the chlorine more strongly than Co(II) and hence the chloride ion gets attached to Cr(III).

(d) Mechanism of photosynthesis by plants has been investigated by 18O. The reaction to formation of sugars from water and carbon dioxide is given below.

$$6CO_2 + 6H_2O \xrightarrow{hv} C_6H_{12}O_6 + 6O_2$$

Formerly, it is believed that the oxygen comes from the CO_2. But, a study with radio isotope, ^{18}O authenticated that O_2 comes from H_2O not from CO_2.

$$6C^{16}O_2 + 6H_2{}^{18}O \xrightarrow{hv} C_6H_{12}O_6 + 6{}^{18}O_2$$

$$6C^{18}O_2 + 6H_2{}^{16}O \xrightarrow{hv} C_6H_{12}{}^{18}O_6 + 6{}^{16}O_2$$

ii) Agriculture: ^{60}Co has been used to study the migratory and breeding habits of insects which appear in swarms and cause havoc. A few insects were dipped in a dilute solution of $CoCl_2$ containing ^{60}Co below the lethal dose. The activity of the radionuclide persisted for more than six months and provided information regarding the hibernating centre and migratory habits of the insects.

Kinetic studies with ^{14}C in the form of $^{14}CO_2$ have been used to understand the nature of photosynthesis. From the rate of intake of ^{32}P under varying external conditions (e.g., soil pH, temperature) and at different stages of growth, one can decide about the optimum utilization of fertilizers. Growing crops, for example, have a higher rate of intake of phosphate fertilizer (60%) compared to grown up crop (30%). Hence, such fertilizer is best applied at the time of sowing.

iii) Medicine and biochemistry: ^{131}I, a γ emitter, has been used in locating brain tumours and malignant thyroid tumours. The iodine is administered to patient as iodo derivative of the dye fluorescein. The dye is preferentially absorbed on the tumour site and the position of the tumour can be located by a counter placed outside the brain. The exact position and area of the tumour is ascertained during surgery by ^{32}P as tracer. Phosphates concentrate more on the tumour site and the short range β-particle from ^{32}P act as informers.

The γ-radiation from ^{60}Co can be used therapeutically in the treatment of deep cancerous growths. ^{51}Cr is used to label red blood corpuscles to study blood changes and blood flow. Obstruction in blood circulation may be diagnosed by injecting sodium chloride containing ^{24}Na into a vein. From the time lag in detecting a signal in a counter placed near the foot of the patient, it is possible to infer any impairment in blood circulation. Similarly, NaI tagged with ^{24}Na may be used to detect any abnormality in the pumping action of heart. ^{24}Na is also used to estimate the total volume of the blood in a body.

iv) Industry: Faults in industrial welding and wear out of pistons may be detected by the high energy γ-rays emitted by ^{60}Co. The isotope may also be used in gauzing sheet thickness and also control the height of the filling during automatic packing of commercial powders. Detection of leaks in underground pipes containing liquids (e.g., petroleum) is also possible with radio-isotopes. The energy of radiation from certain isotopes like ^{90}Sr may be used in charging electrical batteries in submarines and spacecrafts.

Hazards of radiation:

All nuclear and X-ray radiation and also high energy particle are harmful to us. They can severely damage the living cells and the effect may even be transmitted genetically to later generations. Therefore, with increasing use of radioisotope and the development of nuclear power, it is essential to take safety measures and general consciousness.

The radiation may affect the skin and hair. Deep within the body, may induce blood disorders, tumours and damage the bone marrow. The reproductive organs are particularly sensitive to ionizing radiations at any dose. The gene arrangement in the chromosomes is specially affected and the normal gene mutation rate may be adversely and irreversibly increased that producing abnormalities.

Alpha particle have high ionizing power, ionizes helium gas emitted from radioactive elements. The mine workers inhale the gas causing the damage of lung.

Gamma rays and also x-rays are most hazardous due to high penetrating power. Neutrons may give rise to (n, γ) reactions and γ-rays may cause secondary ionisation.

Exposure is measured in units of roentgen, R. It is the quantity of X-ray or gamma radiation that produces 1 esu of charge [$1/(3 \times 10^9)$ C] in 1 cm^3 of dry air at STP.

Safety measures:

1. Use time, distance, shielding, and containment to reduce exposure.
2. Wear dosimeters (e.g., film or TLD badges) if issued.
3. Avoid contact with the contamination.
4. Wear protective clothing that, if contaminated, can be removed.
5. Wash with nonabrasive soap and water any part of the body that may have come in contact with the contamination.
6. Assume that all materials, equipment, and personnel that came in contact with the contamination are contaminated. Radiological monitoring is recommended before leaving the scene.